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Some features of Schrodinger equation:-

(e) From The time independent Schrodinger equation we have

$$\left[\frac{p^2}{2m} + V(x)\right]\psi = E\psi \qquad p = -i\hbar\frac{\partial}{\partial x}$$

$$\Rightarrow \left[-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V\right]\psi = E\psi \longrightarrow (1)$$

As $V = 0$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + E\psi = 0$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2} + \frac{2mE}{\hbar^2}\psi = 0$$

$$\Rightarrow \frac{\partial^2\psi}{\partial x^2} + \frac{p^2}{\hbar^2}\psi = 0 \qquad p = \sqrt{2mE}$$

$$\psi = Ae^{ipx/\hbar} \longrightarrow (11)$$

$E \longrightarrow$ Positive

[If E, (-ve), $p \rightarrow$ imaginary and The solution will blow up for $p \rightarrow \pm\infty$]

∴ Condn is that

(i) E real and +ve

(ii) $p \rightarrow$ real and remain with in $(-\infty \rightarrow +\infty)$

The normalized wave function may be written in the form

$$\psi_p(x) = \frac{1}{\sqrt{2\pi\hbar}} e^{ipx/\hbar} \quad -\alpha < P < \alpha$$

The ~~orthogonality~~ normality condition satisfy by the above equation is

$$\int_{-\infty}^{\infty} \psi_p^*(x)\,\psi_{p'}(x)\,dx = \delta(p-p') \longrightarrow \text{(III)}$$

$\psi_p(x) \longrightarrow$ The simultaneous eigenfunction of the momentum operator.

∴ The complete solution of the time dependent Schrodinger equation can also be written in the form also.

$$\psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \exp\left[i(px - Et)\right]/\hbar \longrightarrow \text{(IV)}$$

For positive value of P, the above equation represents a plane wave propagating along the +ve x direction.

For (−ve) value of P, the above equation represents a plane wave moving along the −ve x dir^n.

· The most general solution of time dependent schrodinger equation is

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} a(p)\, \exp^{i\left(px - \frac{p^2}{2m}t\right)/\hbar}\, dp \longrightarrow (V)$$

↳ The above equation describes a time evoluation of one dimensional wave packet. This can be acheived as follows.

$$\Psi(x,0) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} a(p)\, e^{ipx/\hbar}\, dp \longrightarrow (VI)$$

using inverse Fourier transformation

$$a(p) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \Psi(x,0)\, e^{-ipx/\hbar}\, dx \quad (VII)$$

So knowing the initial wavefunction $\Psi(x,0)$ the momentum amplitude $a(p)$ can be evaluated. And from That the wavefunction $\Psi(x,t)$ for any ~~arb~~ arbitrary time can be evaluated.

— X —

4th

~~Expectation~~ Momentum probability density :

$$\langle P \rangle = \int_{-\infty}^{\infty} \psi^{*}(x,t)\left(-i\hbar\frac{\partial}{\partial x}\right)\psi(x,t)\,dx$$

$$= \int_{-\infty}^{\infty} p\,|a(p)|^{2}\,dp$$ [using the equation (vi) and (vii)].

— x —

Wave packet

Wave packet is an envelope of waves which produces due to the constructive and destructive interference of some waves ~~of~~ with a very small difference in frequencies and amplitude in a dispersive medium.

Wave packets ~~as~~ propagates in the medium with a velocity known as group velocity.